What's Possible - some more thoughts

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I often play with numbers, and accidentally find things that are new to me.

On Sunday, I listened to Vicky's talk about prime numbers and remembered when I had done a similar thing of listing square numbers in four columns, so I doodled a bit.

I found that each square number was either in the 1st or 4th columns only, which made me think.

This means that all square numbers are either exactly divisible by 4, or leave a remainder of 1 when divided by 4.

**Proof:**
Let $k$ be any positive integer and $4k$ be a multiple of 4:

$(4k)^2 = 16k^2 = 4(4k^2)$ divisible by 4, remainder 0
$(4k+1)^2 = 16k^2 + 8k + 1 = 4(4k^2 + 2k)+1$, divisible by 4, remainder 1
$(4k+2)^2 = 16k^2 + 16k + 4 = 4(4k^2 + 4k + 1)$, divisible by 4, remainder 0
$(4k+3)^2 = 16k^2 + 24k + 9 = 4(4k^2 + 6k) + 9$, divisible by 4, remainder 1

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Then I noticed that within a column the gap between squares increases at a linear rate.

For example: in the 1st column, there is 1 blank number (5) between 1 and 9, 3 blank numbers (13, 17, 21) between 9 and 25, 5 blank numbers (29, 33, 37, 41, 45) between 25 and 49 and 7 blank numbers (53, 57, 61, 65, 69, 73, 77) between 49 and 81. I strongly suspect that this pattern will continue....There is a similar pattern of blank numbers in the 4th column, but these appear at intervals of 2, 4, 6, 8, ....again I think this will continue.

What is noticeable is that in the first column, the intervals are consecutive odd numbers, and in the 4th column, consecutive even numbers. There are 4 columns, so each blank represents 4 numbers, look:

1+**8**=9, 9+**16** = 25, 25+**24** = 49, 49+**32** = 81 etc.... the differences between consecutive odd squares are consecutive multiples of 8 (which are divisible by 4). This is because all odd squares have a remainder of 1, when divided by 4.

Similarly, 4+**12**=16, 16+**20**= 36, 36+**28** = 64, 64+**36** = 100 etc.... the differences between consecutive even squares are consecutive multiples of 4, which are NOT divisible by 8 (starting from 12). This is because all even squares have a remainder of 0, when divided by 4. It's just to do with the remainders.

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I then noticed a window pattern, between 4 squares:

{1,4,9,16} framed 1 blank row, {9,16,25,36} framed 2 blank rows, {25,36,49,64} framed 3 blank rows, {49,64,81,100} framed 4 blank rows, {81,100,121,144} framed 5 blank rows, {121,144,169,196} framed 6 blank rows, etc... Again, I think this will continue.

This is the pattern between an even square number, and an odd square number, look:
4+**1(4)+1**=9, 16+**2(4)+1** = 25, 36+**3(4)+1** = 49, 64+**4(4)+1** = 81 etc.... Again, it's because of the remainders.

This was really good fun!!!!!!!!!!

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|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 |
| 29 | 30 | 31 | 32 |
| 33 | 34 | 35 | 36 |
| 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 |
| 45 | 46 | 47 | 48 |
| 49 | 50 | 51 | 52 |
| 53 | 54 | 55 | 56 |
| 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 |
| 65 | 66 | 67 | 68 |
| 69 | 70 | 71 | 72 |
| 73 | 74 | 75 | 76 |
| 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 |
| 85 | 86 | 87 | 88 |
| 89 | 90 | 91 | 92 |
| 93 | 94 | 95 | 96 |
| 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 |
| 105 | 106 | 107 | 108 |
| 109 | 110 | 111 | 112 |
| 113 | 114 | 115 | 116 |
| 117 | 118 | 119 | 120 |
| 121 | 122 | 123 | 124 |
| 125 | 126 | 127 | 128 |
| 129 | 130 | 131 | 132 |
| 133 | 134 | 135 | 136 |
| 137 | 138 | 139 | 140 |
| 141 | 142 | 143 | 144 |
| 145 | 146 | 147 | 148 |
| 149 | 150 | 151 | 152 |
| 153 | 154 | 155 | 156 |
| 157 | 158 | 159 | 160 |
| 161 | 162 | 163 | 164 |
| 165 | 166 | 167 | 168 |
| 169 | 170 | 171 | 172 |
| 173 | 174 | 175 | 176 |
| 177 | 178 | 179 | 180 |
| 181 | 182 | 183 | 184 |
| 185 | 186 | 187 | 188 |
| 189 | 190 | 191 | 192 |
| 192 | 194 | 195 | 196 |