Are We Nearly There?

Each set of coordinates falls along a diagonal D, where D = x + y from each point in the diagonal (x, y). Assume that (1, 1) is the starting point and is not included in the sum.

If the pattern of arrows continues for ever, which point will be the 100th to be visited?

Sum of points:

Knowing that $\frac{n(n+1)}{2}$ and D is the diagonal on which the point lies,

Then the sum S = $\frac{D-1(D-1+1)}{2} - 1$: subtracting 1 because the point (1,- 1) is not included.

$$S = \frac{D-1(D)}{2} - 1$$

$$S = \frac{D-1(D)}{2} - \frac{2}{2}$$

$$S = \frac{D^2 - D - 2}{2}$$

To find the diagonal D:

$$0 = D^2 - D + (-2 - 2S)$$

Round D up to nearest whole number. That is the diagonal for the coordinate.

Last coordinate in the diagonal:

If D is odd, then the diagonal is descending, therefore (D – 1, 1)

If D is even, then the diagonal is ascending, therefore (1, D – 1)

To find new coordinate number m:

If D is odd, then m= $(D - 1 - |S - m|, 1 + |S - m|)$

If D is even, then m = $(1 + |S - m|, D - 1 - |S - m|)$

Are We Nearly There?

100^{th} point:

$$0 = D^2 - D + (-2 - 200)$$

$$0 = D^2 - D - 202$$

Using the Quadratic formula: $D = 14.72 \ldots$

Therefore, the 100^{th} point lies on the 15^{th} diagonal.

The sum of the points at the end of the 15^{th} diagonal

$$S = \frac{D^2 - D - 2}{2}$$

$$S = \frac{15^2 - 15 - 2}{2}$$

$$S = 104$$

Coordinate = $(D - 1 - |S - m|, 1 + |S - m|)$

$$(15 - 1 - |104 - 100|, 1 + |104 - 100|)$$

$$(15 - 1 - 4, 1 + 4)$$

$$(10, 5)$$

** Thus, the 100^{th} point is found at (10, 5)

How many steps will it take to reach (60,40)? Where will the next step take you to?

Each set of coordinates falls along a diagonal D, where D = x + y from each point in the diagonal (x, y). Assume that (1, 1) is the starting point and is not included in the sum.

Therefore, D = 60 + 40 → the point lies on the 100^{th} diagonal.

$$S = \frac{D^2 - D - 2}{2}$$

Are We Nearly There?

$$S = \frac{100^2 - 100 - 2}{2} = 4{,}949 \text{ total points at end of diagonal}$$

Since D is even, the diagonal is ascending, therefore (1, 99) is the 4,949 point.

$$(60 - 1, 99 - 40)$$

$$= (59, 59)$$

The difference between the last step in the diagonal and (60, 40) is 59. Thus,

$$4{,}949 - 59 = 4{,}890$$

** It will take 4,890 steps to reach point (60, 40).

Can you find a method to work out how many steps it takes to get to* any *point?

To find the number of steps *k,* to get to point (x, y).

Find the diagonal that the point lies on, D = x + y.

Use the diagonal D to calculate the sum of the points, S, through the end of the diagonal.

<u>Last coordinate in the diagonal:</u>

If D is odd, then the diagonal is descending, therefore (D – 1, 1)

If D is even, then the diagonal is ascending, therefore (1, D – 1)

Using the wanted point *k* and the last point in the diagonal, find the difference, *d*, between the x and the y- coordinate.

The number of steps to point *k* is the sum, *S* – *d*.

<u>Example:</u> Point (5, 2)

D = 5 + 2 → 7th diagonal.

Are We Nearly There?

$$Sum = \frac{D^2 - D - 2}{2}$$

$$S = \frac{7^2 - 7 - 2}{2} = 20 \text{ steps until point } (6, 1)$$

$$(6 - 5, 2 - 1) = (1,1) \rightarrow d = 1$$

*Number of steps to point (5, 2) is, 20 – 1 = 19 steps.